

# Fermion regularization, fermion measure and axion fields

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Axion fields were originally introduced to control CP violation due to the  $\theta$  term in QCD. Pauli-Villars regularization, or the use of a *parity symmetric* fermion measure, however, preserves CP in the fermion sector. A CP violation arising from the  $\theta$  term can then be neutralized in a natural way by setting  $\theta$  equal to zero.

The discovery of the Higgs boson put a final stamp of confirmation on the standard model of high energy physics, but the axion, which is also very widely expected, has not yet been seen. It is not quite a part of the standard model, but is popularly connected with CP violation. While CP violation is observed in the weak interactions, it is not known to occur in other processes. However, the so-called  $\theta$  term  $\theta F^{\mu\nu} \tilde{F}_{\mu\nu}$  arising from instantons indicates the possibility of CP violation in the strong interactions. As this has not been observed, modifications of QCD have been proposed to suppress the CP violation. The Peccei-Quinn hypothesis [1] introduced an artificial chiral symmetry in an attempt to remove CP violation in the strong interactions. Unfortunately, it leads to the occurrence of a light pseudoscalar particle, the axion [2], which has not been detected in spite of elaborate searches [3]. This may simply be like the delay in the detection of the Higgs particle, but it may not be an experimental failure at all. It may be the case that this is not the right way of explaining the absence of CP violation in the strong interactions. An alternative explanation [4] which does not require axions has indeed been developed. It turns out that a complex quark mass term does not violate CP. We wish to go further here and demonstrate that the axion field [1] does not generate CP violation in the fermion sector and cannot cancel CP violation caused by a  $\theta$  term if a natural regularization or measure is chosen.

While chiral symmetry of the fermion action is broken by a quark mass, a new chiral symmetry can be manufactured by letting a new pseudoscalar field  $\varphi$  absorb the chiral transformation. The mass term is then replaced by

$$\bar{\psi} m e^{i\varphi\gamma_5} \psi, \quad (1)$$

which is invariant under the transformation

$$\psi \rightarrow e^{i\alpha\gamma_5} \psi, \quad \bar{\psi} \rightarrow \bar{\psi} e^{i\alpha\gamma_5}, \quad \varphi \rightarrow \varphi - 2\alpha, \quad (2)$$

which is also a symmetry of the kinetic terms of the action but is anomalous. There have been variations on this theme. The original interaction introduced by Peccei and Quinn [1] was of the form

$$\bar{\psi} [\Phi \frac{1+\gamma_5}{2} + \Phi^\dagger \frac{1-\gamma_5}{2}] \psi, \quad (3)$$

where  $\Phi$  is a complex scalar field with a symmetry breaking potential. The chiral symmetry transformation is

$$\Phi \rightarrow e^{-2i\alpha} \Phi, \quad \psi \rightarrow e^{i\alpha\gamma_5} \psi, \quad \bar{\psi} \rightarrow \bar{\psi} e^{i\alpha\gamma_5}. \quad (4)$$

$\Phi$  may be taken to be of the form  $\rho e^{i\varphi}$ . The amplitude  $\rho$  of the scalar field acquires a vacuum expectation value because of symmetry breaking, which provides a massive boson. The phase  $\varphi$  is the zero mode of the potential and provides the Goldstone boson. This is the axion, which acquires a mass because of the quark masses, but does not appear to exist. How does this mechanism claim to remove P and T violation? It is usually believed that the exponential factor containing the axion field can be absorbed in the fermion fields by a local chiral transformation, which then produces a term  $\varphi F^{\mu\nu} \tilde{F}_{\mu\nu}$  because of the anomaly. The vacuum expectation value of the axion field could then nullify the  $\theta$  in the term  $\theta F^{\mu\nu} \tilde{F}_{\mu\nu}$ , thus removing the CP violation due to  $\theta$ .

That analysis was done before the rôle of measures in anomalies came to be fully appreciated. The anomaly appears in regularized theories, so we shall use an explicit regularization before carrying out the problematic chiral transformation involving the axion field. Does the  $\theta$  term-like structure show up?

A convenient gauge invariant way to regularize the theory is to introduce Pauli-Villars fields. The Lagrangian density

$$\bar{\psi} [i\rlap{D} - m] \psi, \quad (5)$$

where  $D$  is the covariant derivative and involves the gluon fields as we are dealing with strong interactions, goes over to

$$\bar{\psi} [i\rlap{D} - m] \psi + \bar{\chi} [i\rlap{D} - M] \chi. \quad (6)$$

Here,  $\chi$  is a bosonic spinor field, whose mass  $M$  is to be taken to infinity. In the presence of the axion, the coupling has to be introduced:

$$\bar{\psi} [i\rlap{D} - m e^{i\varphi\gamma_5}] \psi + \bar{\chi} [i\rlap{D} - M e^{i\varphi\gamma_5}] \chi. \quad (7)$$

Let us now carry out a local chiral transformation by  $e^{-i\varphi\gamma_5/2}$  on  $\psi, \chi$ . The Jacobian of the measure is trivial in the regularized theory [4]. The phase factors disappear, but the kinetic terms produce derivatives of  $\varphi$ . The effective action arising from fermion integration depends only on the differentiated axion field apart from the gauge field. Therefore in this regularization there is no possibility of a term  $\varphi F^{\mu\nu} \tilde{F}_{\mu\nu}$  in the effective action which could violate CP and nullify the CP violating effect of the  $\theta$  term.

The result goes against the belief that the exponential factor containing the axion field is equivalent to a  $\varphi F^{\mu\nu} \tilde{F}_{\mu\nu}$  term. This is because an explicit regularization has been used here. This regularization does not reproduce the effect

produced by the popular choice of the measure of fermion integration which involves only an implicit regularization and leads to the above term. Does this mean that one of these calculations is wrong? Both are right as far as calculations go, but we shall argue that only one is appropriate in the circumstances.

There is an underlying symmetry. We first note that the axion coupling conserves parity when  $\varphi$  is a pseudoscalar field. Taking symmetry breaking into account, we can still preserve parity if the shifted field  $\varphi' \equiv \varphi - \varphi_0$  is now transformed like a pseudoscalar, where we denote the vacuum expectation value of  $\varphi$  by  $\varphi_0$ . One way of seeing this is to absorb  $\varphi_0$  in new  $\gamma$  matrices

$$\tilde{\gamma}^\mu \equiv \gamma^\mu e^{i\varphi_0\gamma_5}, \quad (8)$$

which satisfy all requirements on  $\gamma$  matrices. Note that products of two  $\gamma$  matrices are unchanged in this redefinition and

$$\bar{\psi}[i\not{D} - me^{i\varphi\gamma_5}]\psi = \psi^\dagger[i\tilde{\gamma}^0\tilde{\gamma}^\mu D_\mu - m\tilde{\gamma}^0 e^{i\varphi'\gamma_5}]\psi. \quad (9)$$

Instead of using new  $\gamma$  matrices, one may equivalently check the invariance of  $\bar{\psi}me^{i\varphi\gamma_5}\psi$  under a new parity transformation

$$\begin{aligned} \psi(\vec{x}) &\rightarrow \gamma^0 e^{i\varphi_0\gamma_5} \psi(-\vec{x}) \\ \bar{\psi}(\vec{x}) &\rightarrow \bar{\psi}(-\vec{x}) \gamma^0 e^{-i\varphi_0\gamma_5}, \end{aligned} \quad (10)$$

which is also consistent with the kinetic part of the action.

The fermion action at the classical level thus has a parity symmetry. Upon quantization, two things can happen. Either there is no regularization which preserves this parity symmetry, in which case one says that the parity is anomalous; or, there are regularizations which preserve the symmetry, in which case they should be used if artefacts are to be avoided. The regularization (7) clearly exhibits the above parity when  $\varphi'$  is transformed like a pseudoscalar and  $\chi$  is transformed in the same way as  $\psi$  under parity. It is therefore the natural regularization.

Instead of an explicit regularization, there is an alternative measure approach with an implicit regularization. The usual fermion integration measure, in this approach, fails to respect the above symmetry. However, there exist choices of the measure which preserve the symmetry [5,6]. These measures are dependent on the axion field. Although a chiral transformation tends to produce a Jacobian factor involving the angle of the chiral rotation, these measures change also when the dependence on the axion field has to be adjusted and the factors connected with these two changes cancel out. This was discussed for a chiral phase in the complex mass term of the fermion, *i.e.*, a constant axion, in [5], and for an axion field in [6]. Instead of expanding the fermion field [7] as

$$\psi = \sum_n a_n \phi_n, \bar{\psi} = \sum_n \bar{a}_n \phi_n^\dagger, \quad (11)$$

where  $\phi_n$  are eigenfunctions of  $\not{D}$ , parity symmetry demands

$$\psi = e^{-\frac{1}{2}i\varphi\gamma_5} \sum_n a_n \phi_n, \bar{\psi} = \sum_n \bar{a}_n \phi_n^\dagger e^{-\frac{1}{2}i\varphi\gamma_5}. \quad (12)$$

The fermion measure  $\prod_n da_n d\bar{a}_n$  is gauge invariant and also parity invariant even when symmetry breaking occurs and  $\varphi_0$  is nonvanishing. The consequence of this measure involving the axion field is that this field is not transferred from the fermion sector of the action to the gauge field sector by the chiral transformation, as there is an additional change in the measure which compensates the gauge field sector. The local chiral transformation replacing  $e^{i\varphi\gamma_5/2}\psi$  by  $\psi$  essentially removes  $\varphi$  from both the measure and the action, except for one residue: the kinetic term of the fermion field is invariant only under global chiral transformations and not local ones, so that a derivative term results:  $\bar{\psi}\gamma^\mu\gamma^5(\partial_\mu\varphi)\psi$ . The upshot is that a  $\theta$  term in the gauge field sector cannot be cancelled by a  $\varphi$  term when this parity symmetric measure is employed. Nonsymmetric measures are unnatural in a technical sense.

One may wonder how much freedom one has in choosing regularizations or measures. Different regularizations have of course been used in the past and it is known that all do not lead to the same result. A key point is that symmetries of the action should be sought to be preserved by the regularization. Thus one is always looking for Lorentz invariant, gauge invariant regularizations for theories with actions having such symmetries. Bypassing this requirement leads to artificial results. It turns out that the regularization can be made parity invariant. In the case of measures the requirement is not so well known because explicit measures are not often used. However, if a symmetry of the action cannot be preserved by any measure, it means that there is an anomaly. When there is no anomaly, there is no reason to go for a measure that needlessly violates a symmetry of the action. Gauge noninvariant or Lorentz violating measures have never been used in theories where the actions have such symmetries. We know that in the case of parity too, an invariant measure exists. If the measure is not constrained to respect the symmetries of the action, it is

possible to violate all symmetries of the action simply by choosing non-invariant measures. In particular, the action with a real mass term can give rise to CP violation through the misuse of a measure not invariant under parity. That is clearly artificial. The moral is that the measure, like the regularization, has to be sought to have the symmetry of the action as far as possible.

There may be an apprehension that a measure is predetermined and not in one's hands. If a field is already quantized, one indeed cannot go and alter the measure, but if one is looking at an action, which is basically a classical concept, a measure has to be chosen. In the case at hand, one must clearly understand where one stands. The quark mass comes not from the fermion sector or the gauge sector but from scalar fields. For completeness, one may start with massless quarks – coupled to scalar fields. The vacuum expectation value of the scalar generates both the mass and the chiral phase  $\varphi_0$ . Note that the *same* phase arises in the mass term and in interaction terms of the quarks with the scalars. So the same parity symmetry exists for both. The quark may be considered to be quantized after that. At this stage a regularization has to be chosen and the Pauli-Villars regularization may be chosen. Alternatively, if an explicit measure is chosen, it is to be chosen with care, – *to preserve its symmetrical shape* .

To sum up, the axion scheme of Peccei and Quinn was invented to engineer the suppression of strong CP violation. We have pointed out a parity symmetry of the classical fermion action including the axion. This symmetry is preserved by explicit regularization and also in the measure approach with implicit regularization unless one chooses to break it. As long as one respects this symmetry, the axion field cannot produce a CP violating  $\theta$ -like term to compensate the CP violation caused by a  $\theta$ . Naturally,  $\theta$  itself may be set equal to zero [4] to avoid CP violation.

What remains finally is a concern about the need for axions. Even if there is no rôle for axion fields in controlling CP violation, they may just happen to exist: this can presumably be determined by experiment and is beyond the reach of field theory.

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